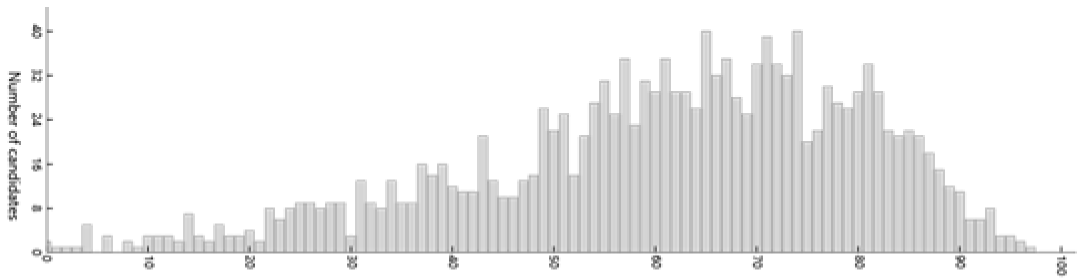
Government of **Western Australia**
School Curriculum and Standards Authority

2021 ATAR course examination report:
Mathematics Specialist

Year	Number who sat	Number of absentees
2021	1503	18
2020	1526	23
2019	1435	32
2018	1546	21

The number of candidates sitting and the number attempting each section of the examination can differ as a result of non-attempts across sections of the examination.

Examination score distribution–Written



Summary
The examination consisted of Section One: Calculator-free and Section Two: Calculator-assumed.

Attempted by 1501 candidates	Mean 60.77%	Max 96.82%	Min 0.00%

Section means were:			
Section One: Calculator-free	Mean 64.63%		
Attempted by 1501 candidates	Mean 22.62(/35)	Max 35.00	Min 0.00
Section Two: Calculator-assumed	Mean 58.68%		
Attempted by 1499 candidates	Mean 38.14(/65)	Max 62.53	Min 0.00

General comments
The 2021 paper gave candidates ample opportunity to demonstrate their ability, as indicated by a mean of 60.77%, in comparison to the 2020 mean of 57.74%. The mean of the Calculator-free section was 64.64%, which contrasted with the Calculator-assumed mean of 58.68%.

There were many standard syllabus questions and correspondingly candidates performed very well: sketching an inverse function (Question 2 part (a)), sketching a rational function (Question 4), integration by substitution (Questions 3 and 8 part (c)), separation of variables (Question 10 part (b)), use of partial fractions (Question 5) and implicit differentiation (Question 18 part (a)).

The paper also contained many questions that examined candidates' conceptual understanding and command of detail. These types of questions proved challenging for many and indicated a less than adequate grasp of the syllabus. This was seen principally in the work in solving a quartic equation in the complex plane (Question 6), the intersection of planes (Question 14) and vectors in three dimensions (Question 16). Candidates' ability to write coherent and meaningful explanations in explaining or justifying mathematics requires further emphasis and practice.

The non-routine questions in this paper permitted the most capable candidates to shine, as indicated by a good number of candidates who scored in excess of 80%.

The distribution of marks, as in 2020, exhibited a large spread, indicated by the standard deviation of 19.28%.

Whilst the paper contained fewer total questions than in 2020, the length of the paper was not considered to be shorter than in previous years. It appeared that most candidates had an opportunity to attempt what they could. The last question (Question 19) had a large number of candidates attempting it, with only a small drop off with the last part (Question 19 part (e)).

Advice for candidates

- Ensure that working is copied correctly from line to line. For example, ensure a negative number does not become positive in the next line.
- Ensure your work or calculations have an obvious sequence or conclusion, enabling the marker to follow your line of thought and to observe a conclusion. Markers cannot be expected to extract meaning from a collection of numbers on the page without any clear conclusion or statement.
- Do not use the word 'it' in explanations. For example, if asked why the inverse of g is not a function, markers do not know what a candidate means when they write 'it is many to one'. You need to be specific in your responses.
- Ensure that the correct units are used in giving answers, particularly in questions asking for a rate of change. Careful reading of the question is central to this.
- Improve the legibility of digits.
- Take care when reading the scale of a graph.
- Use the exact value from the calculator and hence do not truncate decimal places early in a question.

Advice for teachers

- Provide opportunities for students to prove mathematics results. For example, in Question 16 part (c), many candidates thought it was sufficient to report that the CAS calculator said that there was 'no solution', without proving the results.
- Improve the conceptual understanding of the intersection of planes in space, particularly when there is not a unique solution.
- Emphasise the use of correct mathematics vocabulary and provide students with opportunities to explain mathematics concepts.
- Improve students' command of general algebraic tasks. For example, markers witnessed an inability to correctly expand brackets (Question 6 part (a)) or incorrectly assuming distributive laws like $\sqrt{2-\sin^2\theta}=\sqrt{2}-\sin\theta$ (Question 8 part (c)).
- Improve students' knowledge and use of exact trigonometric values.

Comments on specific sections and questions

Section One: Calculator-free (49 Marks)

Candidates performed very well in the Calculator-free section (mean of 64.63%), particularly in working with an inverse function, sketching a rational function and with techniques of integration. There were some excellent efforts in Question 7 part (b) to justify why the separate complex equations had only one common solution.

Weaknesses in routine skills evident in this section were:

- determining the argument for the opposite of a complex number (Question 1 part (a))
- performing manual arithmetic or recalling exact trigonometric values (Question 1 part (b))
- correctly multiplying two complex factors $(z-(2+4i))(z-(2-4i))=(z^2-4z+20)$ (Question 6 part (a))
- solving $z^2-2z+3=0$. It was not expected that candidates would suggest solving $(z-3)(z+1)=0$ (Question 6 part (b)).

Question 1 attempted by 1486 candidates Mean 2.34(/4) Max 4 Min 0
While it was expected that part (a) would have been a straightforward question to begin the paper, it was disappointing that less than half the cohort could correctly state $Arg\left(-z\right)$. Stating the $Arg\left(\bar{z}\right)$ was the common error. Part (b) was answered primarily using Cartesian form, and hence presented many candidates with problems with exact trigonometric values and/or working with fractions. While a vector approach was expected, knowing the exact value for $\cos\left(\frac{2\pi}{3}\right)$ might have been an issue for some. Quite a few candidates who did attempt a vector approach assumed that the diagonal bisected the opposite angles of the parallelogram.

Question 2 attempted by 1499 candidates Mean 7.09(/11) Max 11 Min 0
Candidates performed very well in sketching the graph of the inverse function, indicating that this concept is well understood. In part (b), the explanation offered was often let down by the use of the word ‘it’, with markers not knowing whether the inverse or the original function was being referred to by the use of the ‘it’; candidates must not use this word in any question requiring explanation. Many candidates made correct reference to function g not being one-to-one or being many-to-one. The determination of the defining rule in part (c) was well done. In part (d), given that the rule for $f\left(x\right)$ was provided and an exact value was requested for $g\left(f\left(0\right)\right)$, candidates should not have relied on trying to read the value of $f\left(0\right)$ from the graph. Credit was given when candidates correctly applied their result from $f\left(0\right)$ to determine $g\left(f\left(0\right)\right)$. Part (e) (interpreting the domain and range from the graphs provided) proved to be a challenge, although the more able candidates performed well. While many candidates could explain how this composition would be defined, they could not correctly state the required domain.

Question 3 attempted by 1479 candidates Mean 4.22(/5) Max 5 Min 0
This paper was the first year in which candidates had to formulate their own substitution, and invariably good choices were made. The technique of a change of variable is generally understood, but was undermined by candidates’ poor algebra skills in executing the technique. The factor of 15 in the integrand meant that most candidates should not have needed to work with fractions in the final evaluation.

Question 4 attempted by 1488 candidates Mean 4.36(/5) Max 5 Min 0
The graphing of a rational function involving inclined asymptotes (examined for the first time) was done exceedingly well. Most candidates indicated clearly the inclined asymptote and similarly the labelling of the graph was well done.

Question 5 attempted by 1488 candidates Mean 4.59(/5) Max 5 Min 0
Performance was generally sound on part (a). The use of the absolute value of the natural logarithm in the anti-derivative was well recognised and pleasingly very few did not include the constant of integration. Most were able to deal with the anti-derivative of the $\frac{6x}{x^2+2}$ term well. Some astute candidates knew they did not need to use the absolute value with $\ln\left(x^2+2\right)$ and a few went ahead and employed logarithm properties (not required) to write $\ln\left|(x-2)\left(x^2+2\right)^3\right|$.

Question 6 attempted by 1474 candidates Mean 3.30(/5) Max 5 Min 0
In part (a), it was very disappointing that many candidates could not correctly determine a quadratic factor given that one root had been provided. Candidates struggled to correctly expand $\left(z-(2+4i)\right)\left(z-(2-4i)\right)$, with the common error being to write z^2+20. There was evidence of confusion by what the term 'factor' meant by candidates, who wrote $(2+4i)(2-4i)$ or simply offered up $(2-4i)$ as their answer. Given that this was the first time that a quartic polynomial was given (having no purely real roots), this may have explained the poor performance. In part (b), many decided to use either long division or appropriate use of the distributive property to deduce the other quadratic factor. Unfortunately, those who obtained something like z^2+20 from part (a) usually wrote $P(z)=\left(z^2+20\right)\left(z^2-6z+3\right)$, not ensuring that the expansion gave all the terms of the original polynomial, effectively manufacturing their own secondary polynomial that often had two purely real roots. As further evidence of poor algebra, many candidates believed that $z^2-2z+3=(z+1)(z-3)$ and $\frac{2\pm i\sqrt{8}}{2} = 1\pm i\sqrt{8}$.

Question 7 attempted by 1432 candidates Mean 1.59(/5) Max 5 Min 0
In part (a), almost all candidates showed that they knew how to solve complex equations of the form $z^n=1$, but only a minority could express the 43 roots in terms of some integer k. Precision was required for full marks to be awarded here. Part (b) was a challenging question and correspondingly full marks for an adequately justified solution was rarely awarded. Candidates had to use different parameters to refer to the two equations and do more than simply state that '43 and 47 are prime and have no common factors'.

Question 8 attempted by 1469 candidates Mean 4.19(/9) Max 9 Min 0
Many candidates were able to provide an adequate explanation in part (a) and score full marks. However, $\sqrt{x}>0$, whilst being a true statement, does not lead to deduce that $x>0$. In part (b), the symmetry of the heart curve about $x=0$ was assumed and not stated. In questions such as this, candidates should begin with what will give the total area and then work towards showing why the integrand will involve $\sqrt{2-x^2}$. Many attempted to reverse engineer the result unsuccessfully. In spite of difficulties with parts (a) and (b), candidates performed well with the definite integral evaluation using the given trigonometric substitution. Many candidates appreciated the significance of the result. One candidate made reference to 'τ is the true heart of mathematics' [note that $\tau(tau)=2\pi$].

Section Two: Calculator-assumed (92 Marks)
Performance on the Calculator-assumed section was adequate with a mean of 58.68%, but not as good as the Calculator-free section. This was due to the wider array of concepts being examined, requiring a full understanding of the course. Able candidates were able to demonstrate their sound conceptual grasp.

Weaknesses in routine skills evident in this section were:
- recognising that a set of linear equations has an infinite number of solutions (Question 14 part (c))
- describing the geometric significance of the given equations and solution (Question 14 part (d))
- determining the vector equation of a line or Cartesian equation of a sphere (Question 16 parts (a) and (b))
- proving that two lines in space do not intersect. Stating that a CAS calculator says there is no solution does not constitute a proof (Question 16 part (c)).

Question 9 attempted by 1397 candidates Mean 2.70(/5) Max 5 Min 0
Most candidates appeared to be well prepared for this related rates question and hence performed reasonably well. Errors involved using degrees, applying an incorrect trigonometric ratio, using rounded values early in the solution, or involving the hypotenuse distance in the rate of change. Whilst not required, it was pleasing to see a number of candidates using a trigonometric identity for the derivative of $\tan\theta$.

Question 10 attempted by 1477 candidates Mean 6.06(/8) Max 8 Min 0
In part (a), almost all candidates knew that the derivative needed to be evaluated and a good proportion of these could correctly display the slope on the given diagram. Some indicated a slope much greater than 0.25, apparently through an inability to read the graph scale correctly. The use of the separation of variables technique was well done in part (b) to arrive at $y^2 = \frac{x^2}{2} - x + \frac{1}{4}$. Whether candidates considered positive or negative square roots was not considered in the marking key, with no loss of marks if they went on to consider the positive of the square root. Performance in drawing the solution curve was slightly inferior when compared to Question 11 part (b) in the 2019 paper. A small majority of candidates drew the full solution curve through the required point. About 40% of the cohort drew only the top half of the curve, despite obtaining the equation in part (b) that indicated negative y values were possible.

Question 11 attempted by 1490 candidates Mean 6.66(/9) Max 9 Min 0
Part (a) was done fairly well. The common error was to interpret the argument as $\frac{2\pi}{3}$, i.e. an error in reading the graph. Parts (b) and (c) were quite well done. In part (d), performance was below expectation, with many candidates writing the equality $|z-i| = 2$ and the inequality statement $\frac{5\pi}{6} \le Arg(z) \le \frac{\pi}{6}$, showing a lack of understanding with this statement. It appeared that more practice and understanding of describing loci in the complex plane is required.

Question 12 attempted by 1460 candidates Mean 4.20(/6) Max 6 Min 0
Part (a) was a relatively straightforward question involving simple harmonic motion. Candidates determined the value for n quite well from the given period of motion but had greater difficulty obtaining the correct amplitude A from the maximum speed. This difficulty appeared to stem from an issue with the manipulation of fractions. It was re-assuring that many candidates used the condition $a = -n^2 x$ in part (b) to determine the acceleration without having to know the value of t for when $x = 10$ occurred. Errors occurred often through rounding of previous answers, so that the correct three decimal place value was not obtained.

Question 13 attempted by 1456 candidates Mean 3.65(/5) Max 5 Min 0
Part (a) was done well. In part (b), determining the volume of revolution was reasonably well done. Many candidates left their answer as an integral, assuming they had developed an expression for the volume. It should be clear that a four-mark question requires more, especially when the integral is straightforward, with the expectation that the expression will be completed. In light of previous examination questions, which had asked for an integral expression for a volume and specified that the integral was not to be computed, there should not be any misunderstanding with this type of question. Quite a few used 2π in the expression for the cross-sectional area while others omitted π altogether. Several candidates claimed that only half the circle was 'revolved', so took half of the volume expression.

Question 14 attempted by 1492 candidates Mean 4.28(/10) Max 9.5 Min 0
Most candidates were able to form the required equations using the variables in part (a). In part (b), many candidates did not show how their equations were used in order to arrive at their conclusion. It was disappointing to see candidates merely writing $a + 2c = 44$, without a brief conclusion or any reference to an amount in dollars. The performance on part (c), the solution of three linear equations proved to be disappointing, particularly with candidates having access to a CAS calculator. A large number responded with a matrix of numbers with no conclusion. Many reported that there was no solution – very few candidates understood that the calculator was informing them that there were infinite or many solutions. Having had trouble with part (c), part (d) proved to be problematic. The question asked for the geometric interpretation of both the equations and the solution. Those stating there were infinitely many solutions demonstrated a complete misunderstanding of the question. Geometrically, there were two identical planes (co-incident) and a third non-parallel plane intersecting in a line in space. Part (e) provided a problem-solving element to the examination, with some excellent solutions seen from candidates. From the relationship between the variables (from part (c)), where there were a finite number of possibilities, it was possible to determine a unique solution. Candidates could have used algebra or a table of possible integer values to determine the ticket prices.

Question 15 attempted by 1467 candidates Mean 5.28(/9) Max 9 Min 0
Part (a) was quite a straightforward question and was done quite well. Candidates must state that the sample mean is expected to be normally distributed (since n was sufficiently large) and then declare the parameters of this distribution. Most drew a symmetric distribution in part (b), but were not able to incorporate the low standard deviation of 0.3 into their sketch. With such a low measure of spread, density values greater than 4 and less than 2 had to be displayed as being very low. In part (c), a range of errors was evident. Some did not understand that the question called for a confidence interval to be constructed. Any confidence interval constructed has, as its central purpose, the estimation of reasonable bounds for an unknown population mean. Constructing a confidence interval around the population mean $\mu = 3$ makes no sense when the population parameter is known. Almost all candidates who correctly constructed a confidence interval around the sample mean of 2.1 and observed that the interval did not contain the population mean $\mu = 3$ still failed to give a

sensible interpretation. Admittedly, the question did lead candidates to agree with Anika's conclusion on the grounds that the population mean for the unknown sample was likely to be lower than $\mu = 3$, but it cannot be accepted that we knew that it was drawn from teenagers. Hence, very few scored full marks for a conclusion that rejected Anika's conclusion (for the correct reason).

Question 16 attempted by 1446 candidates Mean 3.78(/8) Max 8 Min 0
The task of determining the vector equation of a line was poorly done in part (a). There were too many errors in determining the direction vector of the line, with many candidates simply giving the direction vector as their answer. In part (b), the Cartesian equation of the sphere question also caused undue difficulties. The idea of a proof continues to cause candidates difficulty, as was the case in part (c). The onus of this question was to show that there were no unique parameter values for λ, μ that gave a common point on each line. Most candidates thought that it was sufficient to report that the CAS calculator had indicated that there was 'no solution'. Algebra had to be employed to prove that there was no unique pair of values. Consequently, only those candidates who were well versed in the requirements of a proof and who could indicate the detail achieved full marks on this question.

Question 17 attempted by 1474 candidates Mean 5.89(/12) Max 12 Min 0
Part (a) was straightforward for two-thirds of the cohort. Many thought that the interval $200 \le \mu \le 600$ had a width of 200. Part (c) was conceptually challenging for most, with many opting to use the formula for sample size, straight from the formula sheet, which was not appropriate. The probability statement required in part (d) proved elusive to many candidates. The different sample size, written in terms of n, clearly caused problems for many. The performance here reflects a lack of conceptual understanding. In part (e), the language used needs to be precise for a response to this type of question. Responses such as 'we don't know' are not sufficient. Answers to part (f) were quite good with part (f)(ii) being more testing.

Question 18 attempted by 1479 candidates Mean 3.76(/6) Max 6 Min 0
The implicit differentiation of the implicitly defined curve was done well. Candidates understood this process and were generally able to obtain the derivative as the subject using appropriate algebra. Errors occurred either through not being able to transcribe correctly from one line of working to another or through incorrect algebraic manipulation. Part (b) proved to be challenging for many. Once it was realised that $y = x^2$ represented the condition for zero slope then candidates were usually able to obtain the required equation for a solution. Three decimal places accuracy was required for full marks (evidence of appropriate use of the CAS calculator).

Question 19 attempted by 1466 candidates Mean 7.74(/14) Max 14 Min 0
Many candidates could not sufficiently show that the integration constant was 740 in part (a) using the condition $x(0) = 100$. Calculating the height above the sloped ground in part (b) was quite well done. Disappointingly, many thought that the question simply required $h(3)$ to be calculated. Quite a few candidates did not attempt part (c). There were many varied approaches taken to this question part, including the use of some physics formulae, but nonetheless candidates did reasonably well on this question. It was pleasing to see candidates think their way through part (d), realising that the position of landing had to be determined first. There was good use of CAS routines here and generally candidates fared well. Part (e) required careful thought; the use of the velocity vector or the derivative of the Cartesian equation could be used, evaluated at particular values. The use of the tangent of an angle to give a slope was a key idea that most seemed to know but a minority realised that the angle for the sloped ground had to be factored in. A wide variety of standards were evident. It was interesting to note that many attempted this question in spite of gaps elsewhere on their paper.